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A Systematic review of the literature on learning and teaching of wave-particle duality**

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Fundamental notions of quantum physics are included in introductory university curricula, in physics teacher training, and even in secondary education, both in Argentina and in various other countries worldwide. Among these fundamental concepts is wave-particle duality (WPD), a key concept and a gateway to the quantum world. This paper presents a critical review of the literature on WPD in specialized journals on physics education and science education at the local, regional, and international context, aiming to gather the most relevant findings regarding its teaching and learning. Preliminary results indicate that these findings can be categorized based on their primary purpose into four categories: Teaching WPD, Learning WPD, Historical and Epistemological Reflections on WPD and Reviews on the Teaching and Learning of Quantum Physics (including WPD).

Keywords: Wave-particle duality, teaching, learning, systematic literature review, secondary education, higher education.

1. Introduction

Wave-particle duality has been one of the key concepts in the historical development of quantum theory. Fundamental concepts of the theory, such as the wave function and its probabilistic interpretations, were developed to address the conceptual difficulty posed by the notion of duality [1, 2]. While current debates in quantum physics are oriented toward issues like quantum entanglement and the measurement problem [3], the notion of duality remains relevant through various interpretations of the formalism that persist today [4–10]. Feynman himself stated that WPD was the essence of quantum physics and perhaps its only mystery [11]. From a conceptual and developmental perspective of quantum physics, WPD can be considered a significant notion – a key concept. Moreover, the problem of interpreting the formalism of the theory remains a current debate, which includes WPD [12]. Different interpretations of quantum theory carry distinct implications for the meaning of duality. These interpretations align with experimental predictions, similar to the Copenhagen interpretation. Today, there is no consensus on the fundamental meaning of duality [3]. This suggests a significant challenge for its integration into the teaching of early 20th-century physics – a challenge that allows for addressing current conceptual reflections on a fundamental level.

On the other hand, official curricular documents for physics teacher training in Argentina, as well as for secondary-level physics education, include contents such as blackbody radiation, the photoelectric effect, the Compton effect, the Bohr atom, the Schrödinger equation, and the double-slit experiment, among others. These topics, developed in the early decades of the 20th century, are intertwined with the notion of duality. WPD thus emerges as the central framework for quantum physics content for the early 20th century, both for teacher training and for secondary education. This local curricular context is similar to the international one [13].

Thirdly, from a pedagogical perspective, quantum physics represents a significant intellectual challenge for students (and teachers) due to the conceptual shifts the theory proposes regarding the nature of matter, which are counterintuitive for most learners. WPD causes confusion, as students' experiences have been with objects that behave either as particles or as waves, but never as both simultaneously [14, 15]. Students tend to combine two different theories in their explanations. For instance, in explaining the double-slit experiment, many students mix the concept of trajectory from classical physics with the wave function concept from quantum physics [15, 16]. Wuttiprom et al. [17] and McKagan et al. [18] reported that students struggle to explain the processes underlying phenomena relevant to duality, such as the double-slit experiment. For example, students were unable to adequately explain interference phenomena with photons or electrons. Many used a collision-like process to explain diffraction instead of

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activating the concept of wave superposition. These difficulties are not overcome by teaching that primarily emphasizes quantum physics formalism; as Garritz [19] suggests, it is beneficial to incorporate the interpretative dimension of the formalism into teaching.

The preceding paragraphs indicate that there are epistemological, curricular, and pedagogical insights that deserve to be paid attention and further explored to address the problem of learning and teaching WPD. The question that emerges and challenges us to go deeper into these insights is: how might instruction on duality be organized, considering not only the difficulties students experience in learning this concept but also the ongoing interpretative debates that still persist around it? The present work represents a first step toward answering that question, offering a critical review of the literature on the learning and teaching of WPD. This review aims not only to highlight key findings regarding WPD teaching and learning but also to integrate historical and interpretative reflections on this notion. These results will guide, in the near future, the design and implementation of a teaching sequence on WPD for physics teacher training and another for secondary education.

An immediate antecedent to this review is the work of Krijtenburg-Lewerissa et al. [20]. Their review focuses on research regarding intuitive conceptions, research tools, teaching strategies, and multimedia applications related to various quantum physics topics. These authors report a review of 75 articles published between 1997 and 2017. The present review complements and updates the study by Krijtenburg-Lewerissa et al. [20], as it covers the period from 2010 to 2024. Additionally, it adds specificity and depth by focusing explicitly on the concept of duality. Finally, it compiles historical and epistemological discussions on WPD, which, we believe, make a substantial contribution to understanding this content and, therefore, its teaching.

The guiding questions for this review are:

RQ1. What significant contributions does the literature report on the teaching of WPD?
RQ2. What are the main contributions that the literature offers regarding the learning of WPD?
RQ3. What connections can be drawn between historical reflections and interpretative issues of WPD and the teaching and learning of this content?

2. Methodology

The criteria for selecting journals involved the convergence of two aspects. The first one relates to the quality of the journals, understood in terms of their inclusion in well-regarded databases and their impact indices. Therefore, we considered titles included in SCOPUS (an open-access database) with an SJR rating. Although there are some controversies about whether impact indices truly reflect the quality of an article, they remain a widely accepted evaluative parameter in the international academic world of scientific publications. An exception to this criterion is the inclusion of the *Revista de Enseñanza de la Física* (not included in SCOPUS but indexed in SCIELO), justified by the fact that it is the only journal on physics education in Argentina, the country from which this study is conducted. The second aspect considered was ensuring a broad geographical distribution of journal titles. We included local titles (Argentina), regional ones (Brazil), and international ones (Europe, USA, Asia). Lastly, when all other criteria were equally met, we prioritized journals focused on physics education over those focused on science education, since WPD is a very specific topic within physics and is likely to have greater representation in journals on physics education.

Under these criteria, the twelve journals included in the review were: *Physical Review Physics Education Research*, *Science & Education*, *International Journal of Science Education*, *Science Education*, *Physics Teacher*, *Physics Education*, *Journal of Research in Science Teaching*, *American Journal of Physics* (Education Section), *Enseñanza de las Ciencias*, *Revista Eureka sobre Enseñanza y Divulgación de las Ciencias*, *Revista Brasileira de Ensino de Física*, and *Revista de Enseñanza de la Física.* These are widely recognized titles within the scientific education research community. Nevertheless, we acknowledge that the criteria for defining and delimiting a literature review always involve some degree of subjectivity and may, therefore, be open to debate.

In the first phase, the titles of all articles from the issues and volumes of the selected journals were reviewed, resulting in a total of 18,693 articles. The search was conducted in the first half of 2024. When the words *quantum* and/or *duality* (or their equivalents in Spanish and Portuguese, where applicable) appeared in the titles, the abstracts were also read. Articles with abstracts that included the word *duality* (or its equivalents in Spanish or Portuguese) were included in the review. When only the word *quantum* appeared in the abstract, the body of the text was checked for the phrase *wave-particle duality.* If this expression was found, the article was included in the review. This selection process yielded 162 articles. In a second stage, the 162 selected articles were read, and those that did not develop WPD were excluded. This procedure resulted in 96 articles for the final review. Two of these articles correspond to previous literature reviews on quantum mechanics contents. The remaining 94 articles were read and analyzed regarding their main contributions to the research questions. In this way, three main contributions were identified (Table 1), leading to the following classification:

- **Teaching of WPD:** These are studies whose primary objective is to investigate the teaching of WPD in different contexts and with various approaches. This category includes articles ranging from studies on specific tools for addressing a

Table 1: Categorization Results. The first two columns include the quartiles and impact indices of the consulted titles.

			Categories		
Q[1]	SJR[2]	Journal	Teaching	Learning	History and Epistemology
Q2	0.524	American Journal of Physics	8	0	7
Q1	1.015	Physical Review Physics Education Research	9	11	0
Q1	1.311	Science & Education	7	2	6
Q1	1.003	International Journal of Science Education	1	3	0
Q1	2.115	Science Education	1	1	1
Q4	0.21	Revista Brasileira de Ensino de Física	7	0	4
Q3	0.514	Physics Teacher	9	0	3
Q3	0.402	Physics Education	6	2	0
Q1	1.882	Journal of Research in Science Teaching	0	1	0
Q2	0.501	Enseñanza de las ciencias	0	2	0
Q3	0.333	Revista Eureka	0	0	0
–	–	Revista de Enseñanza de la Física	1	2	0
			49	24	21

[1]**Q** (quartile) indicates the relative position of the journal within its category in the Scopus ranking. Journals are divided into four quartiles (Q1 to Q4), with Q1 being the highest quartile. The classification used in this table specifically refers to the "Education" category, which belongs to "Social Sciences."

[2]**SJR** (Scimago Journal Rank) is an indicator that measures the visibility and scientific influence of a journal, weighting the citations it receives according to the prestige of the citing journals.

particular moment in a class, such as laboratories, simulations, films, etc., to comprehensive teaching proposals with learning objectives and a set of activities to achieve them.

- **Learning of WPD:** These are articles that report on learning obstacles or learning opportunities in different contexts as students engage in tasks related to WPD. The main focus of this group of articles is on learning, specifically on how students understand the concept of duality.
- **History and Epistemology of WPD:** These are articles that provide historical reflections and interpretations of WPD, which are related to issues in the teaching and learning of several quantum concepts, including WPD.

Finally, two systematic literature reviews on the learning and teaching of quantum physics were surveyed. These articles include WPD as part of their scope. One of them is the previously mentioned work by Krijtenburg-Lewerissa et al. [20]. The other study is by Stadermann et al. [13], which reviews numerous curricula on quantum physics topics from different countries around the world. This study reveals that the content included in secondary school curricula is remarkably similar across the countries analyzed.

3. Results

3.1. Teaching of WPD

In this section, we include studies whose main goal is to investigate the teaching of wave-particle duality (WPD) through didactic resources and proposals in various contexts and with different pedagogical approaches. This category includes studies ranging from the exploration of specific tools such as experiments, simulations, textbooks, etc., to be used at specific moments in a class, to comprehensive teaching proposals that, through specific learning objectives, develop a set of activities designed to achieve them.

3.1.1. Experiments

Experiments refer to experimental setups carried out with physical, tangible instruments that can be manipulated by the teacher or students in the classroom. These characteristics differentiate experiments from simulators (next section). Experimens are used to challenge students and promote learning related to duality. They are utilized to demonstrate either the wave-like or particle-like nature as different possible aspects in which light or electrons can manifest. No studies were found in this category that address the interpretative dimension of the notion of duality.

There is an interesting range of experimental setups, from the Mach-Zehnder interferometer [21], the double-slit experiment with multiple photons or electrons [22], or with a single photon [23, 24], electronic configurations to observe the photoelectric effect [25, 26], to the use of sophisticated instruments like a double-stage microchannel plate in a low-energy electron point projection microscope [27]. Below, we will briefly describe the purpose and/or use of the most commonly referenced experiments in the reviewed studies.

The Mach-Zehnder interferometer is a device used to split a light beam into two separate paths and then recombine them to create an interference pattern. This pattern is used to measure phase differences caused by variations in the paths, allowing the study of physical phenomena such as wave-particle duality. It essentially consists of two beam splitters (semi-transparent mirrors) and two full mirrors, arranged so that the light entering the first beam splitter is divided into two paths. Additionally, the double-slit experiment with photons is used as a starting point to analyze empirical data from experiments requiring more costly equipment, as in the case of the double-slit experiment with electrons [22]. The photoelectric effect is a phenomenon used to demonstrate the particle nature of light. When a material is illuminated with light of a suitable frequency, electrons are emitted from the material. From a wave perspective, light is an electromagnetic wave, but in this case, it behaves as particles called photons. Each photon has an energy amount that depends on its frequency. If a photon has enough energy, when it interacts with an electron in the material, the electron absorbs the photon's energy and is ejected. This experiment is fundamental for understanding that, while light may behave like a wave in some contexts, it also exhibits particle properties in others, as seen in the case of the photoelectric effect.

Among all these setups, interferometers are the most commonly used. Although we found some low-cost experiences that could be easily replicated [22, 26], most laboratories for teaching WPD require detectors or sources that are not easily accessible, are costly, and have a level of complexity that requires considerable time to set up. This aspect is highlighted by Pearson and Jackson [21]:

> *Incorporating these experiments into the curriculum took a substantial investment of time and money. Some of the equipment took a month or more to arrive, and it can require several weeks just to get items unpacked and properly set up. Once everything is assembled, the experiments themselves might take a few weeks to get working satisfactorily.* (Pearson, B.J., & Jackson, D.P., 2010, p 480)

For this reason, many studies have explored the Mach-Zehnder interferometer through simulations or thought experiments [28]. While the studies in this category aim to promote understanding of WPD, they focus on describing the experiment without providing grounded didactic guidelines for its use. Additionally, the vast majority do not present an evaluation of the impact of these resources on students' understanding. An example of this is found in Melhorato and Nicoli [26], where experiences are reported that are easily replicable at any educational level, yet no empirical evidence is provided regarding their effectiveness in enhancing student comprehension.

3.2. Computational simulations

We understand computational simulations as virtual representations of phenomena or activities that allow users to learn by interacting with them [29]. These tools offer the possibility to recreate, visualize, and manipulate various scientific and educational scenarios. Although there are a variety of simulations that address quantum teaching from different approaches [30–32], many of them focus on using the Mach-Zehnder interferometer [33–35]. Through this simulator, students can explore wave-particle duality by observing how light behaves probabilistically when passing through two possible paths. While this simulation provides an interactive and visually appealing experience, we have detected a lack of accompanying pedagogical structure, as exemplified in Cavalcanti, Ostermann, Netto & Lima [34].

Many of these resources are used to promote qualitative discussions [35] and, in few cases, to explain the simulated phenomena using mathematical formalism [30, 34, 36]. Most of these resources are designed for university level [31, 34].

In contrast to experiments, which generally require expensive equipment, most simulations are freely accessible and, in the case of interferometers, allow the use of photons with adjustable incident flux [35]. The ability to use photons in a controlled manner makes this resource a good opportunity for discussing the concept of duality.

As with experiments, these studies about simulations also aim to promote understanding of duality. However, the teaching proposals are not based on previous research from science education. This type of study generally focuses on describing new tools without justifying their design or use based on prior research. In general, they emphasize how to implement the simulation, sometimes investigating students' perceptions after experiencing the proposal, but never analyzing the learning outcomes achieved by the students. Only a few studies aim to validate the didactic proposal [35].

3.2.1. Teaching proposals for DOP

This section groups articles that present structured teaching proposals for addressing WPD at various educational levels. In the literature review of these proposals, certain recurring patterns can be identified that reveal common characteristics.

Firstly, some of these proposals do not center on students' ideas. That is, they do not focus on assessing the progression of students' conceptions throughout the educational process. Instead, they are content centered and tend to prioritize presenting the content accurately from a physics standpoint [37–39].

Another prominent characteristic from the review is that the proposals do not make their epistemological stances explicit [40, 41]. In other words, the interpretative dimension of quantum theory is often implicit rather than a central focus in these proposals. Additionally, a significant number of these proposals do not show what students actually learned. Instead, these studies report on the learning difficulties encountered and the emotional experiences students underwent during the educational process. While this approach reflects a genuine concern for understanding the barriers students face and how these affect their learning, it does not detail how their notions of WPD have evolved as a result of implementing these proposals [33, 42].

On the other hand, studies were found that integrate research findings in science education into the design of their educational proposals. These studies are often based on previous research on students' learning difficulties related to WPD. A notable example is Héraud et al. [43], who explore ways to expand classical particle and wave conceptions to interpret the dual behavior of light, addressing the inherent counterintuitiveness of quantum theory. In this sense, Souza et al. [44] propose an innovative approach to understanding WPD. Rather than viewing quantum objects only as waves or only as particles, they propose a "third option." This approach suggests that quantum objects can behave as both waves and particles, depending on how and from which perspective they are observed. In this way, students can overcome the confusion of having to choose between one or the other, understanding that both behaviors are simultaneously possible. Similarly, Malgieri et al. [41] base their proposal on previous research and employ the "Feynman path integral" approach to teach quantum physics at the secondary level, demonstrating effectiveness in helping students develop a more integrated understanding of WPD, with positive results in learning assessments. In a similar vein, Bitzenbauer [45] developed an educational proposal based on photon experiments. In his proposal, students use simulations with a beam splitter, a device that separates a light beam into two different paths. When viewed in terms of photons, they are sent towards the beam splitter and have a 50% chance of following one path or the other, allowing students to observe the probabilistic nature of quantum phenomena. Through this experience, students gain a better understanding that photons do not behave as particles with defined trajectories. Another example is Scotti di Uccio et al. [46]. They use concept maps, based on science education research, to organize students' thinking about WPD, utilizing evaluation tools based on cognitive theories to validate the impact of their proposal. This study mentions that concept maps are tools that describe how students progress in their understanding of a topic, organizing their knowledge in hierarchical levels, with each level representing a more sophisticated understanding than the previous one. It is discussed how these maps are useful for designing assessments based on cognitive theories and adjusting learning goals according to the expected curricular outcomes.

3.2.2. Approaches based on the nature of science

Another teaching resource involves approaches based on the Nature of Science. These approaches promote understanding of what a quantum object is and also how and in what socio-historical context this notion was developed. Wave-particle duality challenges classical conceptions and demonstrates how scientific theories evolve and adapt to new evidence [47]. By addressing these aspects, students are encouraged to develop a more nuanced and critical view of science, recognizing the uncertainty and complexity inherent in scientific knowledge, as well as the importance of the historical and philosophical context in the formulation and acceptance of theories.

Some of the proposals analyzed and categorized in this section are based on how scientific knowledge related to WPD was constructed [11, 48, 49]. For example, in the work of Lima et al. [11], a didactic approach is proposed that makes De Broglie's ideas on interpreting WPD visible. They note that mathematical formulations do not usually highlight De Broglie's conceptions of WPD. This approach involves breaking down the concept into a sequence of interconnected ideas and results that help build a better understanding of duality. Another example of this type of proposal is that of Énery Melo and Manuel Bächtold [48]. They present a pedagogical tool aimed at teaching the nature of science to future teachers using basic quantum physics content. They employ experimental theater, drawing inspiration from Brecht's theater and Kelly's cyclical learning theory. In their proposal, students adopt different roles to discuss aspects related to the interpretation of WPD.

3.3. Learning of WPD

This category analyzes research whose main focus is the learning of WPD. Although these studies include prior teaching on the concept of duality, the instruction is not the focus of the analyses in this category. Therefore, the objective of this category is to gather and analyze prior research that focus on aspects of WPD learning.

Since the aim of this category is to gather information on possible difficulties or opportunities for learning WPD, we will divide the results into three subgroups: (a) studies that address recurrent learning difficulties in WPD, (b) studies that reported opportunities for learning WPD, and (c) studies where the criterion for defining whether it is a difficulty or an opportunity for learning is not given *per se*, but depends on the pedagogical scaffolding proposed. This idea will be further developed in the respective subsection.

3.3.1. Learning difficulties in WPD

3.3.1.1. Confusion between classical and quantum models

The confusion between classical and quantum models is a widely documented issue in the literature on WPD learning. Various authors point out that students tend to apply classical concepts to quantum situations, leading to an inadequate understanding of WPD. For example, Ayene et al. [14], Singh and Marshman [50], and Bitzenbauer [45] found that, despite having received instruction in quantum mechanics, many high school and university students continue to use classical descriptions in the double-slit experiment. That is, while they may talk about wave interference and superposition, they tend to visualize the electron as a particle following a defined trajectory. This can reinforce the adoption of deterministic interpretations of quantum mechanics over others, such as the Copenhagen Interpretation. Fernández et al. [51] and Sinarcas and Solbes [52] understand that this ontological confusion leads students to interpret electrons and photons as classical particles, resulting in an inadequate understanding of WPD. In line with these findings, Bungum et al. [53] observe that the lack of emphasis on the interpretative dimension of quantum physics perpetuates this confusion, leaving students with an intuitive, classical interpretation of quantum phenomena. Finally, Bøe and Viefers [54] indicate that it is common for students to interpret the wave function using classical ideas, which creates a barrier to understanding the notion of duality.

3.3.1.2. Difficulties in understanding the ontology of quantum objects

Directly related to the previous point is the reported difficulty in understanding the ontology of quantum objects. Baily and Finkelstein [55], Sinarcas and Solbes [52], Fernández et al. [51], and Pereira and Solbes [56] assert that although students use quantum concepts and formalism, they maintain classical thinking that prevents them from understanding that quantum objects are neither waves nor particles but rather a distinct entity that can behave as both, depending on the context. Scotti di Uccio et al. [46] and Krijtenburg-Lewerissa et al. [57] argue that this is partly due to the persistence of quasi-classical reasoning, which complicates the transition towards accepting that quantum objects do not fit into the traditional categories of physics. For example, in the study by Balabanoff et al. [58], it was found that some students describe light as a set of particles traveling in a sinusoidal wave, reflecting an incorrect understanding of WPD.

3.3.1.3. Learning focused on mathematical formalism

It was found that students struggle to connect phenomenological, qualitative, and mathematical levels of explanation when trying to understand wave-particle duality. Bøe and Viefers [54] show that both in high school and university, students face difficulties integrating the wave-like nature of quantum objects with their mathematical representations and experimental manifestations. Phenomenological explanation occurs when students use examples from experiments, such as the double-slit experiment, to explain the wave-like nature of particles, referring to phenomena like the interference pattern. Qualitative explanation happens when students make statements about what particles are and are not, referring to their ontology at the microscopic level, attempting to reconcile classical and quantum characteristics. Finally, mathematical explanation refers to the use of formal tools, such as the wave equation or the wave function, to describe the wave-like nature of quantum objects. This level tends to be the most challenging for students. To address these difficulties, the authors recommend strengthening connections between different levels of explanation through the use of visual resources, group discussions, and the inclusion of historical and philosophical perspectives.

3.3.1.4. Students' preference for realist and deterministic interpretations

Numerous studies report students' preference for adopting realist interpretations, which is closely related to their resistance to abandoning determinism. Ayene et al. [14] indicate that, despite formal instruction, many students continue to use deterministic concepts to explain quantum phenomena, such as WPD. For instance, Baily and Finkelstein [59], and Dini and Hammer [60] found that some students persist in the idea that particles like electrons have defined positions at all times. In other words, they reject the notion that particle trajectories cannot be characterized, even in experiments suggesting otherwise, such as the double-slit experiment with electrons. This realist perspective also manifests in the tendency to interpret probability waves as mere mathematical tools, rather than accepting that they represent non-deterministic quantum states.

3.3.1.5. Belief in sinusoidal trajectories for photons and electrons

McKagan et al. [18] reported a difficulty frequently mentioned in the reviewed articles at both the secondary and university levels: students believe that both photons and electrons travel along sinusoidal trajectories. This seems to be a mental model that helps students make sense of WPD.

3.3.2. Learning opportunities in WPD

3.3.2.1. The value of tentative or ambivalent reasoning

The studies by Hoehn and Finkelstein [61], and Hoehn et al. [62], identify that students engage in tentative or ambivalent reasoning when learning quantum physics, particularly in the context of WPD. This type of reasoning involves students navigating between different ontologies of quantum objects, such as particle and wave descriptions, without these being entirely clear or definitive. Students must manage the coexistence of these ontologies without reaching a fixed conclusion, which can sometimes lead to uncertainty. However, the authors argue that this ambivalence is productive, as it allows students to advance in their understanding of phenomena such as double-slit interference and photon behavior in the Mach-Zehnder interferometer. Tentative reasoning, far from being an obstacle, fosters flexibility and the dynamic use of ontologies, which is key to mastering such complex concepts as WPD.

3.3.2.2. The value of group discussions in understanding wave-particle duality

Small group discussions, in addition to their recognized advantage in fostering collective learning through students' critical contributions, emerge as a key activity for addressing the inherent difficulties in learning WPD. In particular, the studies by Bungum et al. [53] highlight that small group discussions allow students to articulate their conceptual difficulties, deepen their understanding, and formulate new questions, promoting more reflective learning. Pereira and Solbes [56] emphasize the importance of these discussions by enabling students to negotiate different perspectives and representations of WPD, using modes of discourse that contrast the characteristics of classical and quantum physics. This negotiation of perspectives is crucial to avoid confusion between classical and quantum entities and to achieve a more accurate understanding of the nature of quantum objects. Finally, Huseby and Bungum [63] note that, through these discussions, students can clarify fundamental concepts, such as observation in the context of the double-slit experiment, helping them understand observation as an active interaction process rather than simply a passive act of looking.

3.3.2.3. The nature of science as a vehicle for learning wave-particle duality

Several authors suggest that an unparalleled opportunity for learning the concept of duality lies in approaching it through the Nature of Science (NoS). This opportunity is especially valuable in contexts where it is not possible to incorporate the formal aspects of the theory. In this way, students learn both physics and about physics simultaneously. For example, Stadermann and Goedhart [47] highlight that quantum physics provides a highly favorable context for addressing aspects of NoS, such as the role of scientific models, the provisional nature of scientific knowledge, and the existence of controversies in science. Addressing NoS within the context of quantum physics captivates students and engages them more deeply in learning quantum concepts. There is a very productive relationship, in terms of learning, between NoS and quantum physics: NoS serves as a privileged vehicle for learning quantum concepts, and quantum physics provides an excellent setting for exploring key aspects of NoS [64].

3.3.3. Opportunities or difficulties in learning WPD?

3.3.3.1. Variations in students' interpretations according to contexto

What some authors interpret as a difficulty in learning the concept of duality may be considered an opportunity, from another perspective. For example, the diversity of interpretations of WPD among students (and teachers[1]). In the studies by Baily and Finkelstein [55, 59] and Ayene et al. [14], it is concluded that university students form their own (mostly realist) interpretations of the nature of quantum phenomena when explaining the double-slit experiment, especially when teachers are not explicit in addressing the different ontologies of quantum objects. This lack of explicitness can lead students to adopt realist interpretations by default, more in line with the de Broglie-Bohm Interpretation than with the Copenhagen Interpretation. From a pedagogical approach that considers the Copenhagen Interpretation as the only correct one, students' realist thinking would be seen as a difficulty. On the other hand, if the pedagogical strategy includes a discussion of both the Copenhagen Interpretation and the de Broglie-Bohm Interpretation[2] students might identify with one of them and feel part of an ongoing controversy. In this context, these reported reasonings could be considered an opportunity for learning.

3.3.3.2. Variations in the ontologies of quantum objects

Hoehn and Finkelstein [61] observed that the ontologies of quantum objects are not static and vary depending on the context of use. This variability can be a difficulty, as students may be uncertain about which ontology is correct in each situation, potentially causing confusion when interpreting quantum phenomena. However, it could also be viewed as a learning opportunity, as this flexibility allows students to handle different ontologies, facilitating deeper debates related to WPD. More deliberative and dialogic teaching contexts would allow the variability of quantum entities' ontologies to be considered a learning opportunity.

Thus, what is sometimes reported as a difficulty in learning WPD can, from another pedagogical perspective, be considered a learning opportunity. The diversity of interpretations and perspectives opens space for classroom discussions that enrich collective learning and promote more active participation, contributing to a more meaningful construction of knowledge about WPD. This is one of the reasons why we believe that

[1] Dubson et al. [65] found in their study that 48% of teachers interpret the wave function as an information wave, 30% as a matter wave, and the remainder hold a mixed view.

[2] For more details on the Bohm interpretation, see Bohm, D. (1952). *A suggested interpretation of the quantum theory in terms of 'hidden' variables, Physical Review*, 85(2), 166–193.

incorporating the interpretative dimension of WPD into teaching is a positive decision for improving its learning.

3.3.4. Historical reflections and the interpretative dimension of WPD

The findings in this category are historical reconstructions that allow for the analysis and critique of certain ways of presenting WPD-related content in textbooks, as well as reflections on incorporating the interpretative dimension of WPD into teaching.

In relation to the historical reconstructions, Persson [66] analyzes several editions of a widely used modern physics textbook for university courses and finds that, in relation to blackbody radiation, a false historical context is used to follow a content-centered logic. For example, the derivation of the Rayleigh-Jeans law from 1905 is presented as a precursor to Planck's Law from 1900. This tends to perpetuate a quasi-historical myth by situating a product-centered presentation of scientific achievements in a false historical context, thus providing a distorted image of the history and nature of physical knowledge.

Another example is found in the works of Klassen [67], Niaz et al. [68], and Savall Alemany et al. [69], which question the way typical modern physics textbooks present the photoelectric effect as experimental evidence of the particle nature of light. These authors provide a historical analysis revealing that Einstein's 1905 equation, which posited the existence of localized quanta of energy in space that could only be absorbed or emitted as whole units, was not accepted by the scientific community and was dismissed as evidence of the particle nature of light. Moreover, from 1912 to 1915, Millikan devoted all his efforts to experimentally refuting Einstein's proposed equation. However, in 1915 he was able to measure Planck's constant with a 0.5% error margin. Although Millikan conceded that the value obtained for Planck's constant was within the expected range and, therefore, offered evidence in favor of Einstein's equation for the photoelectric effect, he remained convinced that the photoelectric effect needed to be explained by classical electromagnetic theory, which was widely accepted by the scientific community at that time. Einstein himself regarded his 1905 explanation as a provisional idea. At the 1911 Solvay Conference, he stated, "I insist on the provisional nature of this concept (the quantum of radiation), which seems irreconcilable with the experimentally verified consequences of wave theory" [69]. It was not until 1916 that Einstein associated these discrete packets of energy with photons having a well-defined momentum. However, this did not settle the matter for Einstein, who, in a 1951 letter to Besso, wrote, "The whole of 50 years of conscious brooding has not brought me any closer to answering the question: What are light quanta?" [69]. Widely accepted proof of the existence of light quanta at that time[3] only came with the publication of the Compton effect in 1923 [70]. Compton had to discard classical electromagnetic theory and turn to the photon notion to explain the difference between the frequency of incident and scattered radiation when interacting with an electron.

These studies show that the notion of duality was (and remains) difficult to accept and embodies a complexity that is not reflected in most secondary [69] and university [66–68] textbooks. We agree with these authors that concealing the complexity of WPD does not simplify students' learning, on the contrary, it confuses them and predisposes them to mythologize and distort scientific work, which hinders proper understanding. We believe that this complexity must, in some way, be incorporated into the design of WPD teaching.

There is an additional layer of complexity related to the interpretative dimension of WPD. A significant number of articles address the various interpretations of quantum physics and their relationship with the teaching and learning of WPD. An agreement found in these articles is the predominance of the Copenhagen Interpretation in university and secondary textbooks for teaching quantum physics [71]. In contrast to this predominance, some studies suggest the need to go beyond this interpretation, introducing the Pilot Wave Theory developed by de Broglie and Bohm in 1952 [72] or introducing alternative terms to those commonly used in textbooks, such as "quanton" [12].

Norsen [72] proposes an analysis of wave-particle duality through the de Broglie-Bohm Pilot Wave Theory, presenting it as an alternative to the Copenhagen Interpretation. This theory suggests that particle trajectories effectively explain the results observed in scattering and quantum tunneling experiments, eliminating the need to consider wave function collapse. In his article, Norsen demonstrates that these trajectories align with the expectations of traditional quantum mechanics, offering an alternative approach consistent with experimental results. According to the author, this approach enhances the understanding of these complex phenomena by providing a more intuitive and less ambiguous interpretation.

The "quanton" (a term unrecognized by the Copenhagen Interpretation) was proposed by Bunge in 1967 and later by Levy-Leblond in 1988 and 1999 to differentiate a quantum object from any classical object. According to Bunge, "quanton" is the designation that should be used for any system obeying the laws of quantum physics. His aim was to eliminate the idea of wave-particle duality and consider quantum objects on their own terms, neither as waves nor as particles, abandoning analogies with classical physics. Qureshi [73] also uses "quantons" to demonstrate how, in the double-slit experiment, interference patterns (a wave property)

[3] According to Silva [74], the nature of light remains an unresolved issue in the field of Physics.

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and discrete impacts (a particle property) can manifest simultaneously. In this sense, a quanton is an entity that cannot be adequately or exclusively described as either a particle or a wave, but possesses characteristics of both.

On the other hand, Cheong and Song [3] offer an interesting reflexion on the interpretative dimension of WPD, differentiating between consensual and non-consensual aspects of quantum physics. The question they pose is, *How should the interpretative dimension of quantum physics be taught given the lack of consensus in the scientific community?* They present an analytical framework suggesting three different levels of meaning for duality. The first level is phenomenological: there is consensus that phenomena such as the photoelectric effect, the Compton effect, or the double-slit experiment indicate that light possesses both particle and wave properties. This first level is unrelated to quantum concepts such as the wave function and its probabilistic interpretation; it does not involve either formalism or interpretation of duality. Instead, the Einstein-De Broglie relations ($E = h\nu$; $p = h/\lambda$) play a crucial role in deriving the appropriate first-level meaning from related experiments. The first level of meaning results from attempts to interpret experiments without involving formalism or duality interpretation.

The second level involves predicting or explaining phenomena related to duality using core ideas like the wave function, Born's probabilistic interpretation, Schrödinger's equation, the superposition principle, and the uncertainty principle, without making decisions about the reality of theoretical concepts. The core ideas of quantum theory are highly successful in predicting duality phenomena. In this sense, it can be stated with confidence that there is consensus on the utility of theoretical ideas at this second level of meaning for duality.

Finally, there are various interpretations regarding the reality of microscopic objects, the true nature of the measurement process, and the status and role of quantum theory. Different interpretations offer varied assertions about the meaning and nature of duality, constituting the third level of duality's meaning. While the first two levels of interpretation enjoy broad consensus within the scientific community, there is no consensus at the third level. This work clearly illuminates which aspects are consensual (the first two levels) and which are still debated concerning wave-particle duality (the third level).

These authors define a "suspensive perspective" as keeping the first two levels of meaning in quantum theory, which are consensual within the scientific community, clearly separate from the third level of meaning, which pertains to the reality of quantum objects and lacks scientific consensus. They highlight that the suspensive perspective for the double-slit experiment differs significantly from the usual stances in the physics education research community. For example, a study by Mckagan et al. [18] on quantum mechanics included a question on the double-slit experiment. According to them, the correct answer is that each photon passes through both slits and that if a detector is used at one of the slits, the interference pattern is destroyed. However, this answer does not allow for other possible interpretations of the experiment. The suspensive perspective argues that it is not the microscopic object but the wave function that passes through both slits. Therefore, these authors' decision on what constitutes a correct answer is controversial because it includes the Copenhagen interpretation, failing to maintain the distinction between predictive rules and the reality of quantum objects, and thus does not adopt a suspensive perspective on quantum theory.

The previous works highlight the need to make visible, in teaching, aspects that are generally not shown. One of these is the historical dimension of events in the development of duality, and the other pertains to the reality of quantum objects (where there is no consensus) and its relationship with the level of predictive rules (where there is consensus). Bringing attention to the historical dimension of some crucial experiments, as well as the consensus and controversy related to duality, enriches students' conceptual understanding while promoting a more critical and reflective approach to science, recognizing the complexity involved in addressing topics in quantum physics.

4. Conclusions

Back to the first research question, *What are the significant contributions that the literature reports on the teaching of WPD?*, we found similarities with the work of Krijtenburg-Lewerissa et al. [20] concerning the prevalence of certain tools for teaching WPD, such as the use of the Mach-Zehnder interferometer and the double-slit experiment. We also agree that, although these tools are useful for illustrating the probabilistic nature of quantum phenomena, they do not necessarily address the deeper conceptual difficulties faced by students. This will depend on the pedagogical approaches in which they are embedded. Furthermore, we agree that the lack of rigorous evaluations limits the impact of these resources, making validation and replicability in different educational contexts challenging.

With regard to more structured proposals for teaching WPD, we offer new dimensions of analysis compared to the work of Krijtenburg-Lewerissa et al. [20]. In their 2017 review, the presence/absence of qualitative conceptual approaches and mathematical approaches for teaching WPD was analyzed, concluding that the former were scarce and the latter abundant. In our case, although we adhere to that conclusion, we have made progress in investigating the didactic and epistemological foundations of the proposals, which were absent in Krijtenburg-Lewerissa et al.'s [20] review.

Regarding the epistemological dimension of the proposals, we analyzed the extent to which the proposals make explicit, or not, an interpretative framework for experimental results related to WPD. We found that very few proposals make this aspect explicit. Those that do that, are proposals framed within the Nature of Science approach. These teaching proposals include a focus on how scientific knowledge is constructed, creating fertile ground for discussing the different quantum interpretations related to WPD.

Regarding the didactic dimension, some proposals based on didactic research findings have been identified. These are well founded and assessed teaching proposals that involve the use of specific resources, such as literature [43], concept maps [46], or Feynman's multiple paths [41], among others. We value these findings as potentially useful inputs for the design of future teaching and learning sequences on WPD. However, we believe it is necessary to go further towards authentic design-based research, which not only provides results on whether or not the sequence is effective in terms of learning but is also capable of developing humble theories about the ways of learning promoted by such design. This methodology, known as design-based research [75], is increasingly used in the educational field as it allows to build knowledge about teaching and learning of specific topics in real learning contexts, such as classrooms.

Back to the second question, *What are the main contributions that the literature offers concerning the learning of WPD?*, the comparison between our review and the work of Krijtenburg-Lewerissa et al. [20] reveals both significant similarities and differences. Both articles highlight difficulties such as the confusion between classical and quantum models and the preference for deterministic interpretations. In both cases, it is mentioned that students tend to apply classical concepts to quantum situations, as in the double-slit experiment. Additionally, both studies found that students often interpret particle trajectories as well-defined.

However, our review provides new findings that do not appear in the work of Krijtenburg-Lewerissa et al. [20]. One example is the belief in sinusoidal trajectories for photons and electrons. Another finding is that, although both reviews highlight difficulties in integrating different levels of explanation (phenomenological, qualitative, and mathematical), our review reports that the connection between these levels can be facilitated through the use of visual resources and group discussions. We also identify opportunities for learning WPD. We found that the use of tentative or ambivalent reasoning, where students explore descriptions of waves and particles without reaching definitive conclusions, facilitates WPD understanding. Thus, what is sometimes reported as a difficulty in learning WPD can, from another pedagogical perspective, be considered a learning opportunity. Finally, we identified the Nature of Science (NoS) approach as a productive resource for learning WPD, something not discussed in Krijtenburg-Lewerissa et al. [20].

According to the third question: What *connections can be drawn between historical reflections and interpretative issues of WPD and the teaching and learning of this content?* Our contributions are original in the sense that this dimension is not present in the work of Krijtenburg-Lewerissa et al. [20].

On one hand, articles that address historical reconstructions of phenomena related to WPD reveal that these historical reconstructions have been overlooked in the vast majority of textbooks. Most texts organize content around quantum concepts without considering the historical development of WPD. This approach goes over the intrinsic complexity of the knowledge building process to be taught and, therefore, does not align with current discussions on NoS. This observation raises concerns about the advisability of using textbooks in designing teaching proposals based on didactic research findings.

Regarding the interpretative dimension of WPD, it can be concluded that the vast majority of reviewed articles included in the previous two sections do not make the interpretative dimension of quantum physics explicit. Most research on the teaching and learning of WPD does not take an explicit stance on the interpretative dimension, although the majority appear to assume implicitly that the Copenhagen Interpretation is the only valid option. This implicit assumption (also present in popular quantum physics textbooks) promotes, in our view, significant student learning difficulties about WPD. This bias leads students to believe they are not capable of understanding WPD when, at present, there is still ambiguity concerning the reality of quantum objects. Today, several interpretations beyond the Copenhagen Interpretation are accepted, all compatible with experimental results. Incorporating alternative interpretations into WPD teaching remains an outstanding issue that will be considered in designing a WPD teaching-learning sequence, which is the next step in this research.

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