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CHEMICAL SCIENCES

Modeling Particle Sedimentation in Drilling Fluids using Gaussian Process Regression: A Computational Approach for Industrial Applications

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Abstract: Sedimentation is a fundamental unit operation in the oil industry. For characterizing this phenomenon, understanding the concentration and velocity profiles of particles is essential. Simulating this process requires integrating phenomenological (differential) models with constitutive (algebraic) equations. The inherent complexity of these models often makes simulation time a limiting factor for practical applications. This study aims to utilize Gaussian Process Regression to regularize experimental data points and predict the behavior of volumetric concentration profiles of solids in a particle sedimentation model in Br-Mul drilling fluid, used in the petroleum industry. Concentration was monitored over a 500-day period as a function of height and time, using the Gamma-ray attenuation technique. The numerical results obtained demonstrate that the proposed methodology was able to achieve good estimates for the concentration profiles at different points in the monitored domain. However, it is emphasized that the fluctuation of experimental points can result in physically unfeasible profiles. Finally, it is important to note that the computational cost required for this approach is significantly lower than that typically required by algebraic-differential model simulations.

Key words: Metamodeling, Sedimentation, Drilling Fluid, Gaussian Process Regression.

INTRODUCTION

Computational simulation has emerged as one of the most significant pillars in engineering and related fields. This is due to the necessity of representing natural phenomena for purposes of simulation, design, control, and research (Bequette 1998). Over the past decades, the advent of high-performance processors at relatively low costs, combined with the development and refinement of numerical techniques, has allowed for the tackling of increasingly realistic problems. With the use of computational simulators, it is possible to reproduce phenomena with a certain degree of accuracy and minimize the number of experiments required for a given application (Fontana 2018).

Naturally, the problems representing natural phenomena are inherently governed by Algebraic-Differential Equations. This type of model associates an Ordinary or Partial Differential Equation with algebraic relations known as Constitutive Equations. The phenomena represented by differential equations arise from the application of conservation laws, such as mass, energy, and momentum. Constitutive equations, on the other hand, represent empirical relations typically characterized through experimental data and are valid only within certain operational ranges (Bird & Stewart 2007).

The computational cost of solving complex models can be high. Recently, the scientific community has shown growing interest in using approximate models, or metamodels. Generally, the idea behind this approach is to evaluate an algebraic equation instead of a differential equation, which, in practice, means reducing the time required to simulate the model. Thus, if the phenomenon under analysis can be represented by a purely algebraic equation, its applicability in different contexts can be enhanced, gaining in terms of processing time, allowing, for example, for online optimization of processes and analysis of uncertainties associated with these phenomena.

In the context of the algebraic representation of natural phenomena, the specialized literature presents two major branches: deterministic and stochastic (i.e., non-deterministic) techniques (Kou & Gao 2014). In the first, a polynomial is generally considered a (deterministic) model for representing the phenomenon. In the second, a combination of different models (deterministic and stochastic) is considered for representing the profiles of interest. In both strategies, the goal is to determine the parameters of the mathematical model considered so that it can represent the behavior of the phenomenological system under analysis. Among the main examples of the first class is the Response Surface Methodology (RSM) (Montgomery 2000). Conversely, Kriging interpolation and Gaussian Process (GP) regression serve as good examples of this second class (Chilès & Desassis 2018). As described by Schulz et al. (2018), both Kriging and GP regression are statistical techniques used for modeling and predicting spatial data or point-referenced data, but with distinct origins and foundations. Kriging returns a point prediction and a measure of uncertainty (standard deviation) for each predicted point. GP regression, however, is a non-parametric Bayesian approach for curve-fitting problems (Gershman & Blei 2012). This approach can capture a wide variety of relationships between inputs and outputs by utilizing a theoretically infinite number of parameters, allowing the data to determine the level of complexity through Bayesian inference (Williams 1998).

Sedimentation is a commonly employed process in chemical engineering, particularly in the mining and oil industries. It involves the separation of heterogeneous mixtures consisting of two or more phases, where, under the influence of gravity, the mixture is gradually separated based on differences in density. In this process, the denser phase settles at the bottom of the container, while the lighter phases form layers above, with the least dense phase positioned at the surface (Matos 2014). This process is applied to mixtures of suspended solids, where solid particles are dispersed in a liquid, making knowledge of particle concentration over time and height extremely important (Arouca 2007).

Mathematically, sedimentation is modeled by combining phenomenological models, based on Mixture Theory from Continuum Mechanics, with constitutive equations that describe relationships between process variables derived from experimental data (Arouca 2007). Due to the inherent complexity of these models, simulation time can hinder applications where rapid evaluation of such models is necessary.

Although the objective in many applications is phase separation, this is not the case when it comes to drilling fluids for oil wells. These fluids are specifically formulated to meet the demands of the drilled region. They contain insoluble solids intrinsic to the formulation and incorporate cuttings generated from the breakdown of the rock formation (Thomas 2001). The drilling fluid is circulated through the well to perform various functions, including transporting cuttings to the surface for cleaning. However, if the fluid flow is interrupted due to corrective or preventive operational stops, the

suspended particles naturally tend to settle, potentially causing equipment damage and hindering process resumption (Gandelman & Pinto 2009, Oliveira et al. 2007).

Another situation where sedimentation occurs in the petroleum industry is in the fluid that remains confined after the wellbore cementing. In this context, phase separation leads to sediment formation. This region is subject to thermal energy transfer during oil extraction, which leads to increased pressure in the confinement area (Jinge et al. 2015, Wang et al. 2021). To mitigate this pressure, a relief zone is designed during wellbore cementing, taking into account sediment height and compaction. Utilizing experimental results for representing this phenomenon and obtaining information on the sedimentation rate, sediment compaction, and height is crucial for the petroleum industry, as it provides greater safety for operators, protects the environment, and is economically advantageous (Thomas 2001).

Given the above, this study aims to apply Gaussian Process (GP) regression to simulate the volumetric concentration profiles of solids during particle sedimentation in the Br-Mul drilling fluid employed during oil well drilling. For this purpose, a set of experimental data was obtained by monitoring the volumetric concentration of solids as a function of height and time over 500 days using the Gamma-ray attenuation technique (Damasceno 1992). In this case, for each time instance, metamodels are generated using GP regression for the set of experimental points obtained during particle sedimentation in the Br-Mul fluid.

This work is structured as follows. Section 2 presents a description of the experimental procedure adopted for monitoring the volumetric concentration of solids as a function of height and time. Section 3 provides brief reviews of metamodeling techniques, highlighting Response Surface Methodology (RSM), Kriging, and GP regression. The proposed methodology and the results obtained are presented in Sections 4 and 5, respectively. Finally, the conclusions are described in the last section.

Description of the Experimental Procedure

For the purpose of this study, the drilling fluid known as Br-Mul (density of 1,145.9 kg/m³) was monitored using the gamma-ray attenuation technique over a period of 500 days. The solids were obtained using a FANN retort kit, model 210463, with a 50 mL capacity. Subsequently, they were analyzed using helium gas pycnometry on a Micromeritics Gas Pycnometer, model AccuPyc 1330, resulting in a specific mass of 2,709 kg/m³ and an initial solid concentration in the suspension of approximately 14%. The characterization of the solid size was conducted with a laser granulometer, the Malvern Mastersizer MicroPlus MAF 5001®, yielding the following values: $D_{0.1}$ = 3.008 mm, $D_{0.5}$ = 40.803 mm, and $D_{0.9}$ = 232.247 mm, where $D_{0.1}$, $D_{0.5}$, and $D_{0.9}$ represent the particle diameters at 10%, 50%, and 90% of the cumulative volume distribution, respectively. Rheologically, the material behaved as a time-dependent pseudoplastic non-Newtonian fluid.

To monitor the variation in the volumetric concentration of solids, an experimental unit comprised of a metal structure with a lifting platform was used. This unit contained an Americium-241 gamma-ray source and a detection system, which included a thallium-activated NaI scintillator crystal, a high-voltage source, a preamplifier, an amplifier, a channel analyzer, a pulse counter, and a computer equipped with Maestro® software. To ensure the proper functioning of the experimental unit, the following values were used (Arouca 2007): 900 V for the high-voltage source and a radioisotope energy range of 500 to 800 mV.

A test tube was positioned between the gamma-ray source and the detection system, allowing for the monitoring of different heights of the study fluid through the vertical movement of the platform. The results, measured as radiation intensity (I), were corrected for the system's resolution time (t_m= 240±50 μs) and converted into volumetric concentration of solids using the equations presented in the work by Gardner & Ely-Jr (1967):

$$R = \frac{I}{1 - t_m I} \tag{1}$$

$$\ln\left(\frac{R_0}{R}\right) = \delta \varepsilon_s \tag{2}$$

where R and R_0 represent the corrected pulse counts reaching the detection system after passing through a solution with and without suspended solids, respectively, ε_s is the volumetric concentration of solids, and δ is a calibration constant obtained with the experimental points of the gamma-ray attenuation technique in the situations of homogeneous suspension (start of sedimentation) and at the end of sedimentation in the clarified liquid region. In this study, the value obtained for δ was 0.205.

Metamodeling Techniques

As previously mentioned, to represent the various phenomena that can be encountered in nature, both differential and algebraic models may be considered. The major advantage of algebraic models is that if the phenomenon under analysis can be represented by a simpler model, its evaluation is practically instantaneous. On the other hand, if the model to be evaluated is differential, the time required for its evaluation is significantly higher than that required for evaluating a purely algebraic model.

During the parameter estimation process in a model, it can be overfitted, underfitted, or it may have good representativeness, as illustrated in Figure 1 for a set of experimental points (Cressie 1993).

In this figure, it is possible to observe that in an underfitted model, it does not fit the data adequately, meaning it fails to find patterns in the data and ignores a large portion of this set. Thus, it can be said that its predictive capability is very limited. An overfitted model, however, is one where the model fits through all the points. In this case, since experimental points inherently present errors, the model captures not the trend but the exact locations without considering the errors. Finally, a well-represented model is one that successfully identifies all patterns characterizing the data while avoiding random points and unnecessary patterns known as noise.

As described earlier, inherent errors arise during the acquisition of experimental data points. In this case, to avoid the chance of obtaining an overfitted model, one can regularize to smooth the set of experimental points (Cressie 1993). Thus, in the case of an overfitted and complex model, regularization can be used to enable better generalization, meaning the proposed model can capture the details of the phenomenon analyzed without increasing its mathematical complexity (Rennen 2009). As highlighted by Tian & Zhang (2022), a regularization technique aims to improve a model's generalization ability by imposing a penalty on a finite dataset or even with inadequate iteration.

Below is a summarized presentation of the most commonly employed approaches for representing phenomena found in nature using metamodels.

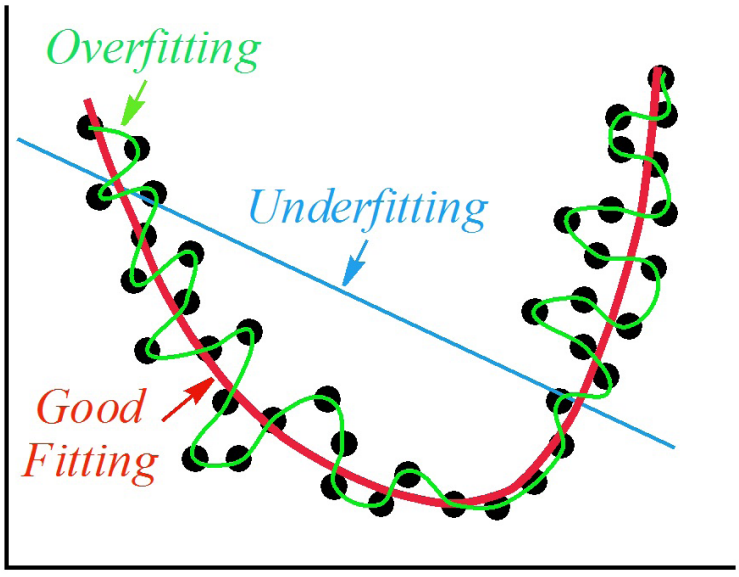

Figure 1. Differences between well-fitted, overfitted, and underfitted models.

Response Surface Methodology (RSM)

RSM is a collection of mathematical and statistical tools used in research to represent phenomena observed in nature (Montgomery 2000). It consists of a series of design and analysis experiments aimed at understanding how independent variables affect dependent variables (Box & Hunter 1978). According to Montgomery (2000), RSM seeks to establish a description of how a response is affected by several factors within a region of interest, as well as to study and explore the relationship between response variables at extreme values. Its goal is to identify and explore areas near maximum or minimum responses, depending on the research focus.

In general, the following equations are employed for the mathematical representation of a given phenomenon:

$$Y = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \Phi \tag{3}$$

$$Y = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \sum_{ii=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \sum_{i}^{k} \beta_{ij} x_i x_j + \Phi \tag{4}$$

where Y is the response being analyzed, x is the vector of independent variables, β is the vector of parameters to be determined through the resolution of an optimization problem, and Φ is the residual.

As seen in these equations, a polynomial is considered where the influences of interaction terms can be analyzed as a model for representing experimental points related to a given natural phenomenon. In this case, the main advantage is the simplicity of the model used. However, this simplicity may not be sufficient to achieve good agreement between experimental points and those simulated by this polynomial model (Montgomery 2000).

Kriging

To enhance the predictive capability of a polynomial model, mining engineer Danie G. Krige (Krige 1951) proposed combining polynomial models with stochastic models for geostatistical analysis. This

approach, known as the Kriging interpolation method, was refined a few years later by Matheron (1963). However, it was only with the work of Sacks et al. (1989) that this methodology gained prominence as a strategy for modeling engineering systems.

In general terms, the Kriging interpolation method involves treating the proposed model as capable of representing a stochastic process (Forrester et al. 2008, Gaspar et al. 2014, Hussein & Deb 2016). For this reason, the mathematical model for approximation using Kriging consists of a combination of two contributions:

$$Y(x) = F(x) + Z(x) \tag{5}$$

where $Y(x)$ is the unknown deterministic response, $F(x)$ is a simpler function (usually polynomial, as described in the previous subsection) of x, and $Z(x)$ is the stochastic contribution with zero mean, variance σ^2, and non-zero covariance, known as correlation functions. These functions can be classified into two groups: those with a parabolic behavior near the origin (Gaussian, Cubic, and Spline) and those with a linear behavior near the origin (Exponential, Linear, and Spherical). According to Lophaven et al. (2002), the selection of a correlation function should be guided by the characteristics of the phenomenon under analysis. For example, if the phenomenon to be modeled is continuously differentiable, the correlation function will likely have a parabolic behavior near the origin, suggesting that Gaussian models, cubic approximations, or splines should be candidates. On the other hand, if the phenomenon shows linear behavior near the origin, an exponential, linear, or spherical correlation would be natural candidates for representing the process under analysis.

In a stochastic model, variations in the experimental points are partially random. Thus, modeling a stochastic process in relation to experimental data provides insight into how this function might behave and how it tends to change as new points with different quantities in each coordinate are introduced (Lophaven et al. 2002).

From a mathematical standpoint, in the stochastic process, it is assumed that the errors are dependent, meaning the correlation between the errors is related to the distance between the corresponding points. This distance can therefore be modeled using the following expression:

$$d(x^i, x^j) = \sum_{l=1}^{K} \theta_l \left| x_l^i - x_l^j \right|^{p_l} \tag{6}$$

where θ and p are parameter vectors that must be determined for each variable that needs to be adjusted. It is important to note that the parameter vector p is related to the smoothness with which the correlation between two points decays as they are separated (Lophaven et al. 2002).

In the Kriging interpolation method, the random variables are correlated through the following relationship:

$$d(x^i, x^j) = \exp\left(-\sum_{l=1}^{K} \theta_l \left| x_l^i - x_l^j \right|^{p_l} \right) \tag{7}$$

From Eq. (7), it is possible to construct a correlation matrix of all the samples:

$$\Psi = \begin{pmatrix} \operatorname{cor}\left(\beta(x^1), \beta(x^1)\right) & \cdots & \operatorname{cor}\left(\beta(x^1), \beta(x^n)\right) \\ \vdots & \ddots & \vdots \\ \operatorname{cor}\left(\beta(x^n), \beta(x^1)\right) & \vdots & \operatorname{cor}\left(\beta(x^n), \beta(x^n)\right) \end{pmatrix} \tag{8}$$

where:

$$\mathrm{cor}(\beta,\beta) = \sigma^2\Psi \tag{9}$$

The matrix Ψ allows a set of correlated random variables to be described in matrix form. More details on the mathematical development of the Kriging interpolation method can be found in Lophaven et al. (2002).

Gaussian Process Regression

The vast majority of statistical models used for curve fitting are based on, or aim to work with, normal distribution due to its mathematical simplicity and ease of handling (Gershman & Blei 2012). Among the families of stochastic processes most commonly used for this purpose, Gaussian Processes (GP) regression stand out as they are structured around the normal distribution and, consequently, offer two essential properties (Schulz et al. 2018). The first property is that a GP regression can be characterized by its first-order (mean) and second-order moments (variance), also known as the kernel, which determines the structure of functions in the function space of the Gaussian Process. The second property is that the best predictor of a GP at an unobserved location is based on the multivariate normal probability distribution function (Gershman & Blei 2012).

As described earlier, GP regression is a Bayesian non-parametric approach used for curve fitting problems (Gershman & Blei 2012). In this context, GP regression involves estimating a function for arbitrary inputs, given a prior training set, by considering an extension of the multivariate Gaussian distribution (Mackay 1998, Rasmussen & Williams 2006).

From a mathematical perspective, a GP regression defines a distribution over functions such that observations of the outputs at any selected points follow a joint Gaussian (multivariate) distribution. Formally, a GP regression is defined as a collection of random variables, any finite subset of which has a joint Gaussian distribution.

In GP, it is assumed that an output y from a function f with input X can be written as (Schulz et al. 2018):

$$y(X) = f(X) + \omega \tag{10}$$

where the noise ω follows a normal distribution with mean 0 and variance σ^2.

In this case, in GP regression, unlike in linear regression, it is assumed that the signal noise ω is a random variable following a particular distribution. This distribution is subjective in the sense that it reflects our uncertainty about the function. As discussed by Schulz et al. (2018), uncertainty about f can be reduced by observing the function's output at different input points. Thus, ω reflects the inherent randomness in the observations. In summary, in GP regression, the function $f(X)$ is given as:

$$f(X) = GP(m(X), k(X,X')) \tag{11}$$

where the mean function $m(X)$ reflects the expected value of the function at input X, i.e., the average of all functions in the distribution evaluated at input X. As mentioned earlier, this mean function is often defined as zero to make the inference determined solely by the covariance function (Schulz et al. 2018).

The function k is known as the GP kernel (Jäkel et al. 2007). As noted by Schulz et al. (2018), the kernel function can be chosen based on assumptions such as smoothness and expected patterns for the data. In this case, a good assumption is that the correlation between two points decays with the distance between them. This means that closer points should behave more similarly than points that are farther apart. Based on this idea, a good choice for the kernel is the radial basis function, defined as (Schulz et al. 2018):

$$k(X, X') = \sigma_f^2 \exp\left(-\frac{\|X - X'\|^2}{2\lambda^2}\right) \tag{12}$$

where λ (length scale) and σ_f^2 (signal variance) are hyperparameters that must be chosen to increase or decrease the a priori correlation between points and, consequently, the variability of the resulting function. In summary, by selecting these parameters appropriately, the data can be modeled more smoothly, i.e., regularized (Mackay 1998, Rasmussen & Williams 2006, Schulz et al. 2018). It is important to note that these hyperparameters were determined through the optimization of the Gaussian Process's marginal likelihood (Cui et al. 2021).

Differences between Kriging and Gaussian Process Regression

As presented earlier, both Kriging and Gaussian Process (GP) regression are strategies used for nonlinear modeling of spatial or point-referenced data. There are similarities between the two approaches, but also significant differences (Cui et al. 2021, Christianson et al. 2023).

Perspective

Kriging is derived from a frequentist perspective, aiming to obtain the best unbiased linear estimator for the target variable from a dataset. In contrast, GP regression is based on a Bayesian perspective, aiming to sample the posterior distribution of Gaussian Processes given a dataset.

Dimensionality

Regarding the dimensionality of the system to be adjusted, Kriging is designed for spatial analysis in 2D/3D space. GP regression, however, has no dimensional restrictions and is naturally suited for high-dimensional spaces.

Fitting Functions

In terms of fitting functions, although it is possible to propose more complex functions, most Kriging tools allow only a few simple functions like spherical, exponential, and power functions. On the other hand, GP offers a more flexible and powerful non-parametric model with different kernel strategies, enabling even the combination of more complex functions. Additionally, the GP approach is more automated; hyperparameters are determined by maximizing the marginal likelihood of the selected data kernels. Finally, it is important to emphasize the regularization of data through the application of GP regression.

METHODOLOGY

The methodology proposed in this work for simulating the concentration profiles during the sedimentation of the Br-Mul drilling fluid consists of two stages: the regularization of experimental points and the prediction of concentration profiles using Gaussian Process (GP) regression. It is important to highlight that, due to experimental errors inherent in any laboratory procedure, it is necessary to smooth (regularize) the collected data to capture the trend of the physical phenomenon accurately. The second phase is employed to predict the behavior of the phenomenon at positions not experimentally measured.

The proposed methodology is illustrated in Figure 2, with the steps summarized as follows:

1) **Experimental Data Points**: The experimental points considered in this work represent the volumetric concentration during particle sedimentation in the Br-Mul fluid. The volumetric concentration of solids was monitored as a function of height and time over a period of 500 days using the Gamma-ray attenuation technique.
2) **First Stage**: For each position of the sedimentation column, the volumetric concentration of solids over time was regularized (smoothed) using GP regression.
3) **Second Stage**: To estimate the volumetric concentration profiles of solids for different positions in the sedimentation column at each time point, GP regression was employed again. At the end of this process, a metamodel is generated as a function of the respective heights, defined for each time point considered.

RESULTS AND DISCUSSION

To evaluate the proposed methodology, the study was conducted using a paraffin-based Br-Mul drilling fluid that had previously been used in the field. Therefore, besides the solids from the fluid's formulation, it also contained cuttings resulting from the rock formation breaking during well drilling.

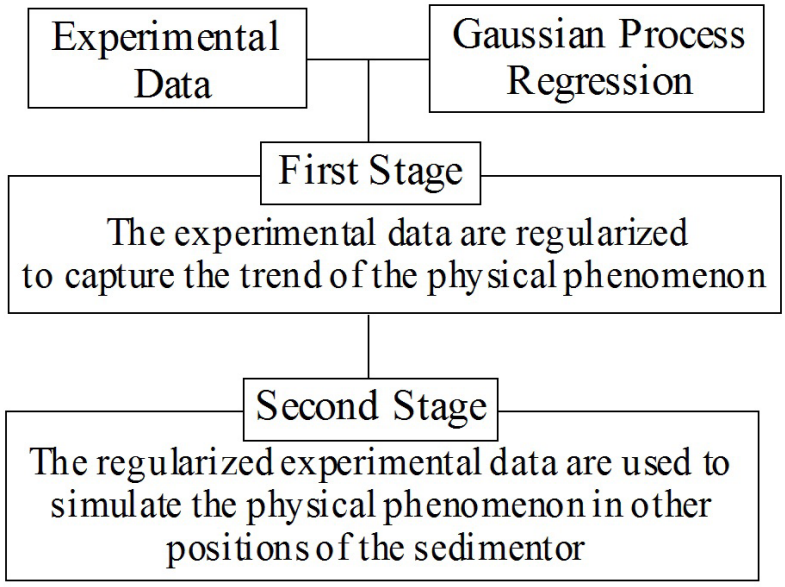

Figure 2. Flowchart of the proposed methodology.

This fluid was provided by Petrobras for this research. Although the exact formula is confidential, it was found to exhibit pseudoplastic and thixotropic behavior, characteristics expected for fluids used in oil well drilling. The density and specific gravity indicate the presence of a barite weighting agent, which is commonly used in such fluids.

In the experiment, the fluid, after being homogenized, was poured into a cylindrical test glassware and positioned between the gamma-ray source and the detection system. Since this fluid had been formulated and used in the field, its gravitational sedimentation occurred slowly, allowing the volumetric concentration variation of solids at predefined heights ([0.5, 1, 2, 3, 4, 6, 8, 12, 16, 18, 20] cm above the base) to be monitored once a week.

The results are presented in two sections. The first section shows the estimated profiles without regularizing the experimental points, using Kriging with an association between a polynomial expression and an exponential-type correlation for this purpose. The second section considers the regularization of the experimental points using GP regression. The BFGS (Broyden-Fletcher-Goldfarb-Shanno) method (Liu & Nocedal 1989) was employed to determine the hyperparameters, with a tolerance of the order of 10^{-6} and the following ranges for the hyperparameters: $10^{-4} \leq \sigma_f, \lambda \leq 10^3$ (defined from preliminary runs). The results were obtained considering a 99% confidence interval.

Unregularized Experimental Points

In Figure 3, the volumetric concentration profiles of solids for each height over time are shown, considering the unregularized points (original data without any treatment) using the Kriging method.

In this figure, it can be observed that, from a mathematical standpoint, the Kriging method was able to provide good estimates for the volumetric concentration profiles. Furthermore, it is noted that these results are overfitted; that is, each metamodel exactly passes through the respective sets of experimental points (resulting in a coefficient of determination - R^2 - equal to one for each fit). Although this may seem advantageous, from a physical perspective, an overfitted model is not ideal

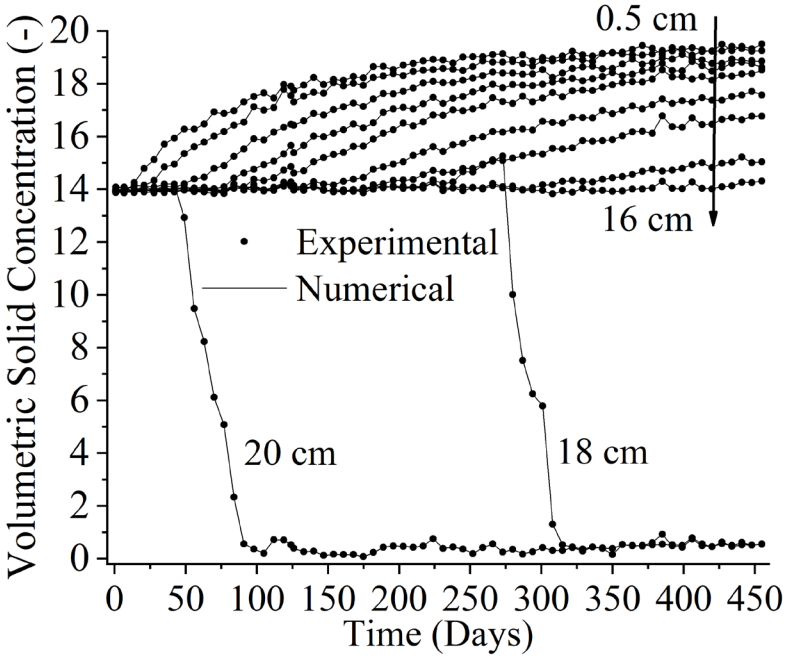

Figure 3. Volumetric concentration of solids versus sedimentation time considering unregularized data (heights = [0.5, 1, 2, 3, 4, 6, 8, 12, 16, 18, 20] cm).

in this case, because the resulting profiles are not naturally smooth due to the presence of errors in the experimental points.

Regularized Experimental Points

As observed in the previous section, although the parameter estimation technique results in a good approximation for the volumetric concentration profiles of solids, it is necessary to regularize the points for the purpose of smoothing these profiles, as they cannot be used for reproducing the sedimentation phenomenon. Table I presents the values of the hyperparameters (σ_f *e* λ), the objective function, the number of iterations, and the R^2 for each height of the sedimentation column considering Gaussian Process regression. This table shows that, for all positions of the sedimentation column, GP converged to an optimal value with few iterations. Moreover, the R^2 value is close to one, which is an excellent indicator of the quality of the solution found.

In Figure 4, the volumetric concentration profiles at each height over time are shown, regularized using GP regression. In this case, unlike what was observed in Figure 3, the curves are visibly smoothed, capturing the trend of the data rather than forcing the metamodels to pass through the experimental points, thus avoiding overfitting.

With the experimental points regularized, the next step is applying GP regression to estimate the metamodels for different heights (not experimentally measured) over time. These estimates are presented in Figure 5. It is observed that the prediction of concentration profiles for most points within the range [0.5, 20] cm does not correspond to what is expected physically; oscillatory profiles are noted, which are not observed during the sedimentation phenomenon. Additionally, negative concentration values, which are not physically acceptable, were obtained. Mathematically, this unexpected behavior is due to oscillations in the experimental points. Consequently, the metamodels at these heights exhibit oscillatory behavior that propagates along the sedimentation column. This factor can be observed in Figure 6 for heights of 12 and 16 cm, respectively.

Table I. Hyperparameters and performance metrics for GP regression at various sedimentation column heights.

Height (cm)	σ_f	λ	Objective Function	Iterations	R^2
0.5	2.9435	106.8272	0.0133	30	0.9934
1	1.9492	103.8888	0.0119	27	0.9951
2	1.5284	80.4201	0.0068	32	0.9976
3	1.7053	129.9633	0.0121	26	0.9959
4	1.5743	112.6137	0.0096	29	0.9965
6	1.1936	93.6154	0.0070	29	0.9962
8	1.2018	186.4544	0.0108	32	0.9895
12	0.4121	110.7665	0.0057	31	0.9548
16	0.0749	20.2552	0.0052	29	0.9465
18	5.2423	38.5334	0.2056	30	0.9948
20	4.5843	39.4199	0.0668	30	0.9967

In this case, as seen in Figure 6, there are fluctuations in the experimental points that cause the volumetric concentration profiles of solids at 12 and 16 cm to cross. Moreover, the metamodel generated for the height of 16 cm, at least up to 150 days, overestimates that obtained for the position of 12 cm. In practice, attempting to predict concentration profiles for heights near this region results in an amplification of this oscillatory behavior, leading to concentration profiles that do not align with physical expectations, as highlighted in Figure 5.

To assess the impact of oscillatory behavior, Figure 7 shows the volumetric concentration profiles of solids for heights between 0.5 and 8 cm, with oscillatory data points removed. It can be seen there is an agreement between the estimated profiles and what is physically expected, demonstrating that the oscillatory behavior of the experimental points is responsible for obtaining physically unviable results. It is important to note that, from a physical perspective, it was not possible to predict the

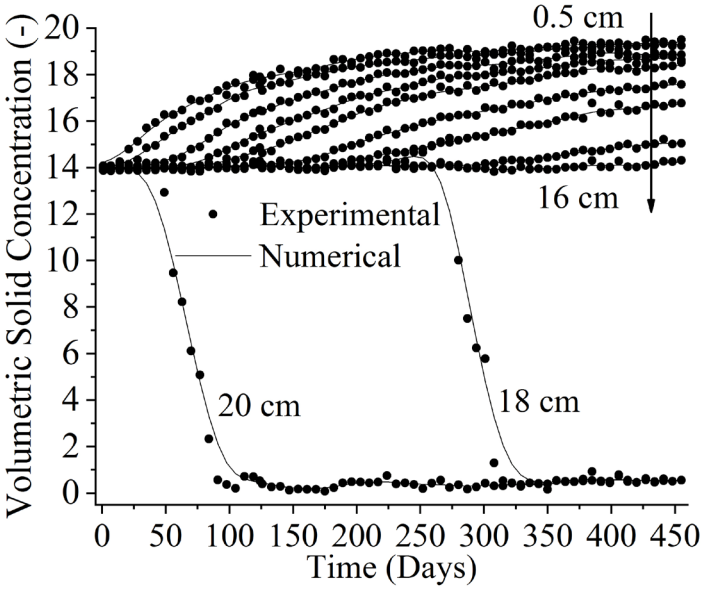

Figure 4. Volumetric concentration profiles of solids versus sedimentation time considering regularized data.

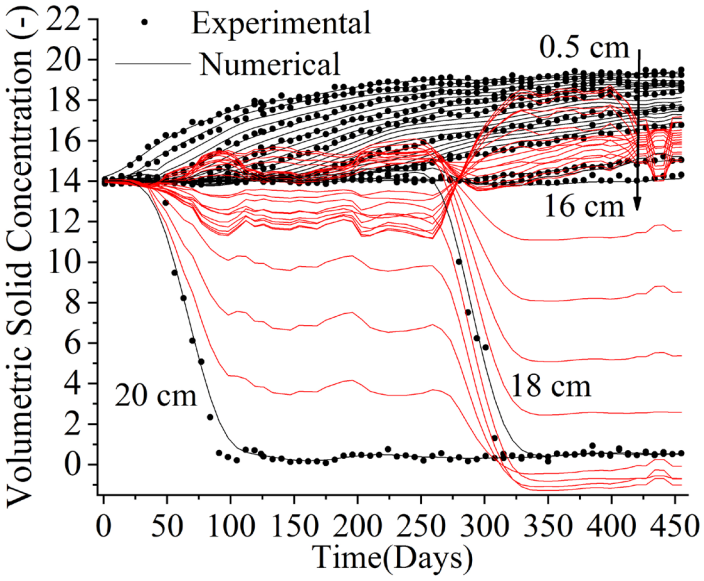

Figure 5. Prediction of volumetric concentration of solids versus sedimentation time for regularized data at intermediate heights between 0.5 and 20 cm.

behavior of the volumetric concentration of solids for heights above 8 cm. In this region, with time, a clarified phase naturally forms, where fewer positions were monitored over time. On the other hand, in the region close to the maximum concentration (base of the sedimentation column), it was possible to predict volumetric concentration profiles of solids. Furthermore, due to the regularization of experimental points, these profiles show good agreement with mathematical expectations.

CONCLUSIONS

This study aimed to obtain the volumetric concentration profiles of solids during the sedimentation of particles in the Br-Mul fluid through a unidimensional and transient experiment, as well as to determine the volumetric concentration profiles at positions not experimentally measured, using

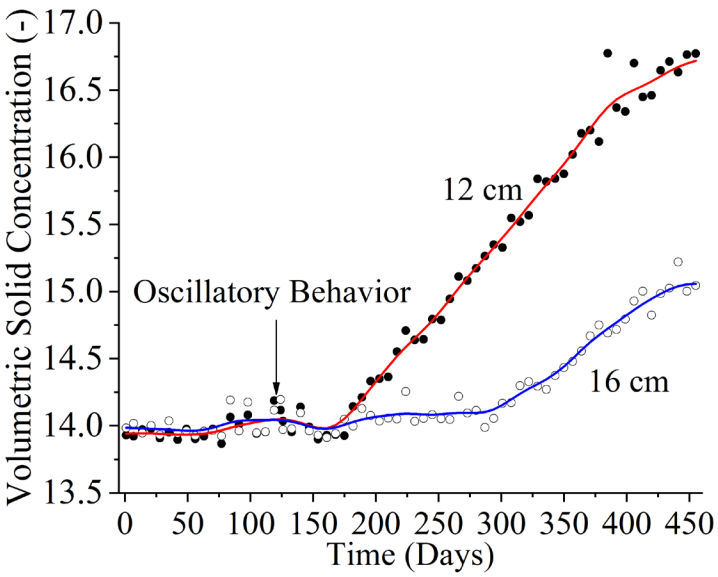

Figure 6. Prediction of volumetric concentration of solids versus sedimentation time for regularized data at heights of 12 and 16 cm.

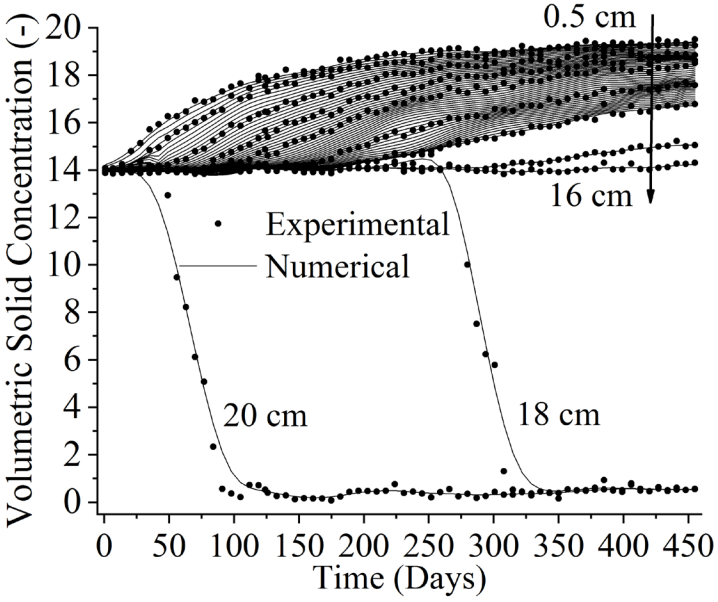

Figure 7. Prediction of volumetric concentration of solids versus sedimentation time for regularized data at intermediate heights between 0.5 and 8 cm.

Gaussian Process (GP) regression. For this purpose, the volumetric concentration of solids was monitored as a function of height and time over approximately 500 days through the application of the Gamma-ray attenuation technique. To achieve smoother metamodels, GP regression was employed to regularize the experimental points, thereby accurately capturing the trend of the physical phenomenon.

The findings indicate that: i) Regularizing experimental points significantly aids in smoothing laboratory profiles and refining proposed models; ii) Estimating volumetric concentration profiles for heights above 8 cm was challenging due to oscillations in the experimental data and metamodels; iii) Regularized experimental points provided accurate estimates for volumetric concentration profiles.

It is important to highlight that a primary advantage of the proposed methodology is the reduction in computational cost required to evaluate a traditionally costly model from a mathematical standpoint, as it involves an algebraic-differential equation system. Additionally, the quality of the experimental results obtained was crucial for ensuring physically coherent profiles for heights below 8 cm. Finally, despite the difficulties encountered for heights greater than 8 cm, the results obtained contribute significantly to the modeling of this complex engineering system.

As suggestions for future work, it is intended to advance the study of sedimentation considering metamodels, as well as utilize artificial intelligence techniques for refining and predicting concentration profiles.

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